Graphs and Units of Slope and Y-intercept

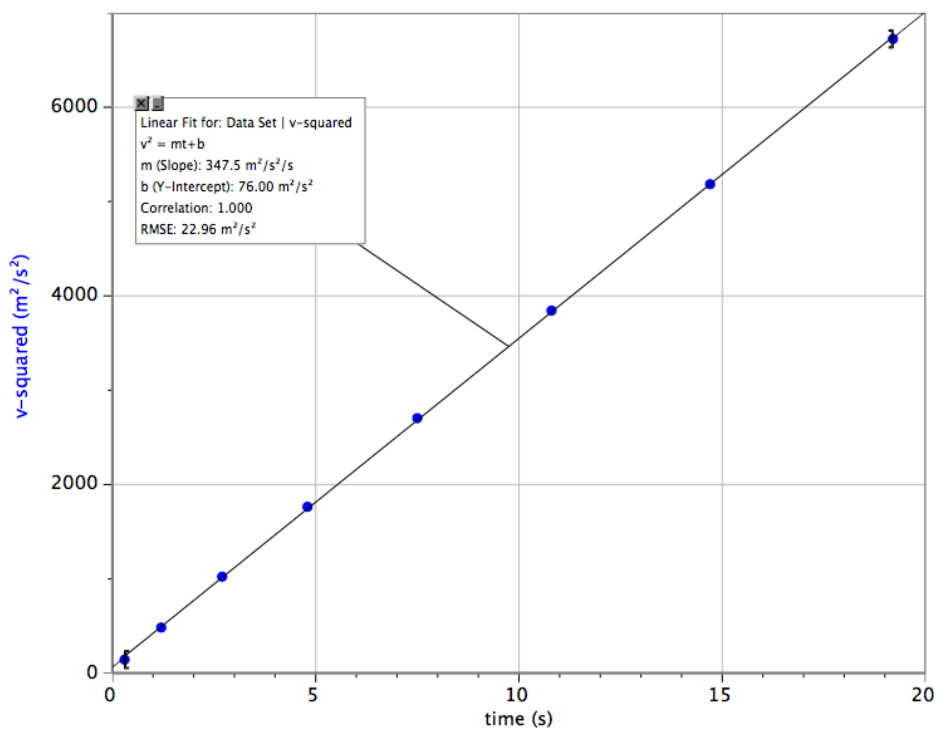
After students linearize a graph, they know that they can find the relationship between the two variables on the X- and Y-axes. They merely use the equation for a straight line $y = mx + b$ where m is slope and b the Y-intercept. But what are the units of the slope and Y-intercept? One answer is that the units of the slope are the units of $\Delta y/\Delta x$ and the units of the Y-intercept are the units of the Y-axis. The next best way to answer this question is to use information provided by the graphing program *Logger Pro.*

t (s)	**v (m/s)**
0.3	12
1.2	22
2.7	32
4.8	42
7.5	52
10.8	62
14.7	72
19.2	82

Say that students are given the data to the left for time t (expressed in seconds, s) and velocity v (expressed in meters per second, m/s). A graph is made plotting (t, v) and a right-opening parabola results. *Logger Pro's* "Data: New Calculated Column…" tool is used to square the velocity term. The data are then re-plotted (t, v^2) and a linear relationship results as is shown in the graph below.

Carefully examine the linear fit for the data set involving v-squared. (See the box within the graph.) Note carefully that the now linear relationship is given explicitly as $v^2 = mt + b$. Note the fact that m (Slope) is given as 347.5.$m^2/s^2/s$. The units can be simplified by using the well-known relationship

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc} \quad : \quad \frac{\frac{m^2}{s^2}}{\frac{s}{1}} = \frac{m^2 * 1}{s^2 * s} = \frac{m^2}{s^3}$$



Now, b (the Y-intercept) is given by *Logger Pro* as 76.00 m^2/s^2.

The <u>physical</u> form of the relationship is then properly written as follows, including units:

$$v^2 = 347.5\frac{m^2}{s^3}t + 76.00\frac{m^2}{s^2}$$

Note that when time is inserted into the equation (say t = 4.500s) the units work out properly for determining v which is expressed in m/s. That is,

$$v^2 = 347.5\frac{m^2}{s^3}(4.500s) + 76.00\frac{m^2}{s^2}$$

$$v^2 = 1564.\frac{m^2}{s^2} + 76.00\frac{m^2}{s^2}$$

$$v^2 = 1640.\frac{m^2}{s^2}$$

$$v = \sqrt{v^2} = \sqrt{1640.\frac{m^2}{s^2}} = 40.50\frac{m}{s}$$